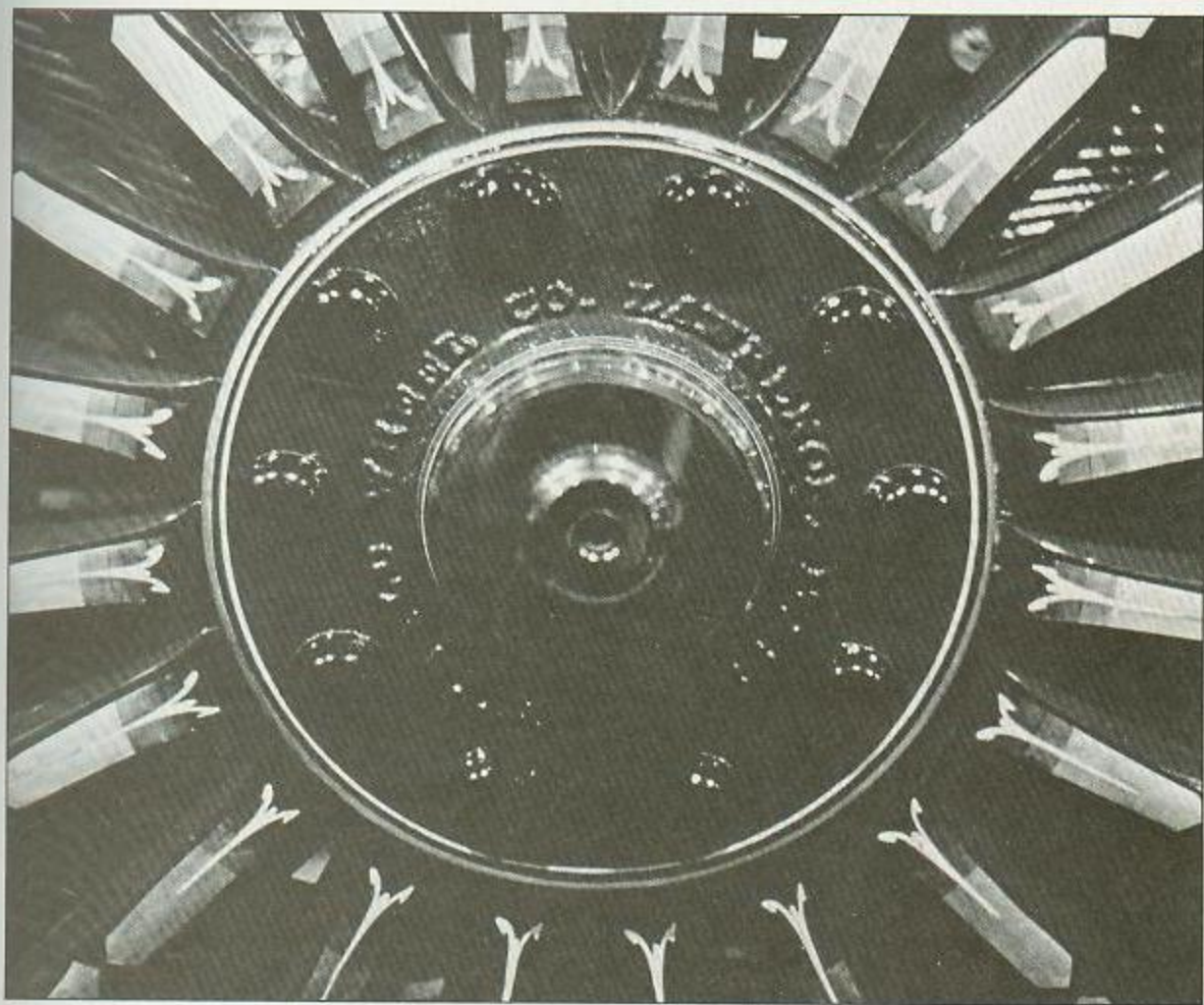

CHAPTER

1

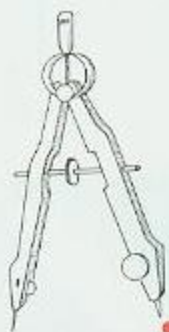
- 1-1 Point, Line, Plane, and Space 10
- 1-2 Relations Among Points, Lines, and Planes 12
- 1-3 Some Basic Geometric Figures 16
- 1-4 Segments and Angles—Congruence and
Measurement 20
- 1-5 Segment and Angle Bisectors 24
- 1-6 Perpendicular Lines and Planes 28
- 1-7 Polygons 32
- Important Ideas* 36 *Review* 37 *Test* 38
- Problem Solving Techniques**
 - Draw a Diagram 39
- Geometry In Our World**
 - Interior Design: Tessellations 40

Definitions and Constructions



1-1 Point, Line, Plane, and Space

How would you describe a point? a line? a plane? space? These four ideas are very important in the study of geometry. Yet we will not define point, line, and plane. Instead, we will look at objects that suggest these ideas.



A point as a part of a physical object

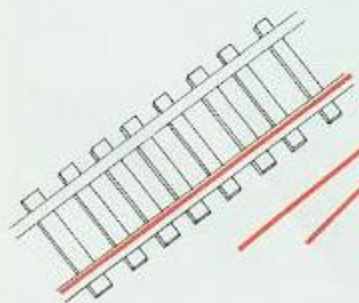


A point as the smallest dot you can draw

POINT

location, no length, width or height

A point is an idea, or abstraction. Since a point cannot be defined using simpler terms, it is an undefined term.



A line as part of a physical situation

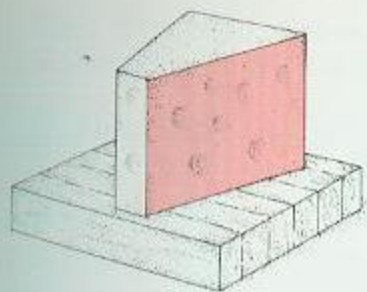


A line as the thinnest streak you can draw

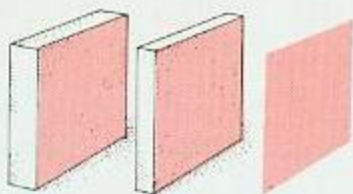
LINE

unlimited length, straight, no thickness, no endpoints

A line is an idea or abstraction. Since a line cannot be defined using simpler terms it is an undefined term.



A plane as a part of a physical object



A plane as the thinnest slice you can cut

PLANE

no boundary, continues in all directions, flat, no thickness

A plane is an idea, or abstraction. Since a plane cannot be defined using simpler terms, it is an undefined term.



Points on, inside, and outside the balloon



Space as that which remains after the balloon has been removed

SPACE

no boundary, length, width, and height

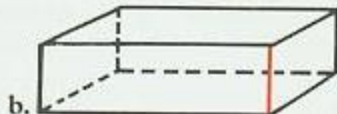
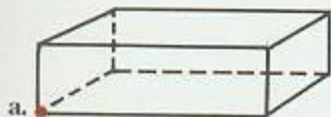
Space is an idea, or abstraction.

Definition 1-1

Space is the set of all points.

EXERCISES

1. Indicate whether the red portion of each of these figures suggests point, line, plane, or space.



2. Name five objects that are shaped so that part of them suggests the idea of point. Identify the specific part of each object.
3. Name three objects or physical situations that illustrate the idea of a line or part of a line.
4. Name five objects that are shaped so that part of them suggests the idea of plane. Identify the specific part of each object.
5. Name three objects, like the balloon, that can be used to suggest the idea of space.

1-2 Relations Among Points, Lines, and Planes

We draw dots on paper to represent points. Capital letters beside them name the points. We call them *point A*, *point B*, and *point C*.

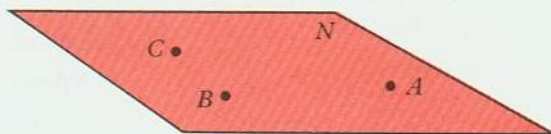


We can think of a line as a set of points. By labeling a pair of points, we can name the line in terms of two points. For example, points *A* and *B* are on the line, so we call it *line AB*. We assume that only one line goes through both *A* and *B*. Another way of saying this is, *Two points determine a line*. Sometimes a line is labeled by using one small letter. Here line *AB* can also be called *line l*.



A plane can also be thought of as a set of points. A plane is named either by placing a single letter by the plane or by naming a set of three points on the plane that are not all on one line. We say *plane N* or *plane ABC*.

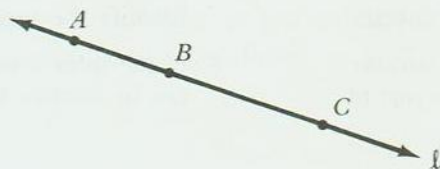
Points *A*, *B*, and *C* are on plane *N*.



We assume that only one plane contains the three points. We say that three points not all on one line determine a plane.

When thinking of line *l* as a set of points, we can say point *A* is on line *l* and point *A* is an *element of* line *l* to describe the same situation. We can also say *line l* contains point *A*.

If *A*, *B*, and *C* are points on line *l*, as shown in the figure, we say point *B* is *between* points *A* and *C*. If *A*, *B*, and *C* are not all on the same line, we do not use *between* to describe their relationship.



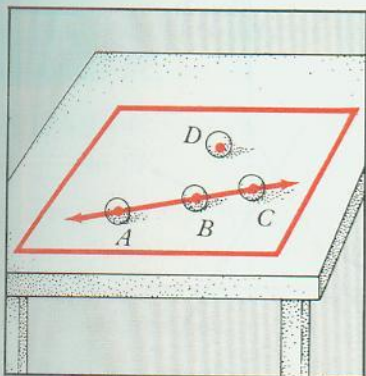
Point *B* is between points *A* and *C*.

Some of the basic relationships for points and lines in a plane are described below using models, symbols, and definitions.

Physical Model,
Figure

Description, Symbol

Definition



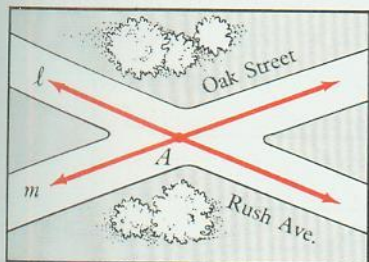
A , B , and C are *collinear*. A , D , and C are *noncollinear*. A , B , C , and D all lie in the same plane. They are *coplanar* points. Points not all in the same plane are *noncoplanar* points.

Definition 1-2

Collinear points are points that lie on the same line.

Definition 1-3

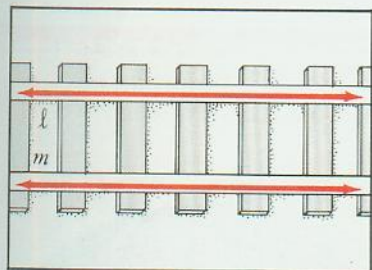
Coplanar points are points that all lie in one plane.



Lines ℓ and m intersect at point A .

Definition 1-4

Intersecting lines are two lines with a point in common.

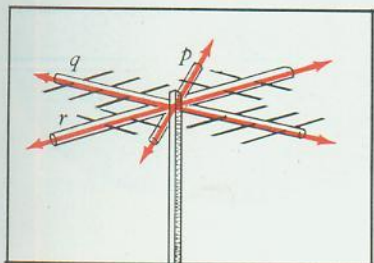


Lines ℓ and m do not have a common point. ℓ is *parallel* to m .

We write: $\ell \parallel m$

Definition 1-5

Parallel lines are lines in the same plane that do not intersect.



Lines p , q , and r have exactly one point in common. They are *concurrent lines*.

Definition 1-6

Concurrent lines are three or more coplanar lines that have a point in common.

EXERCISES

A.

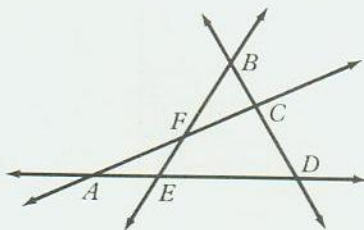
1. Draw three points that are collinear.
2. Trace the set of dots shown and use a straightedge to draw a line through sets of three or more collinear points.



(Ex. 2)

Exercises 3, 4, and 5 refer to the figure on the right.

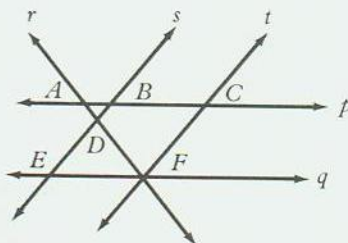
3. Name sets of three collinear points.
4. Name sets of three noncollinear points.
5. Name four points, no three of which are collinear.



(Exs. 3-5)

Exercises 6-8 refer to the figure on the right. (If lines appear to be parallel, you may assume that they are.)

6. List three pairs of intersecting lines.
7. List three concurrent lines.
8. List all pairs of parallel lines.
9. Draw four concurrent lines.

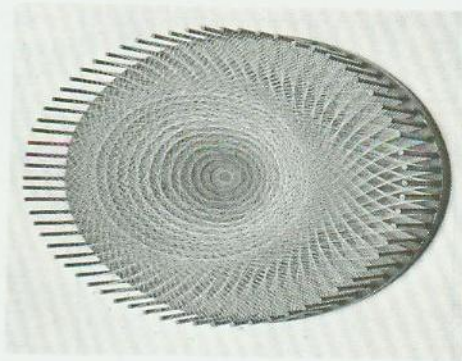


(Exs. 6-8)

Activity

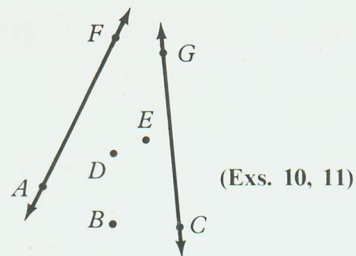
Curve-stitching designs or *string sculpture* is the creation of interesting designs made entirely with straight lines of string or thread. The designs can be very simple or quite complex.

To get an idea how a *string design* is made, trace the angle below and mark it as shown. Use a fine point pen or pencil and connect points with the same number.

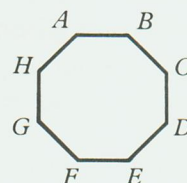


B.

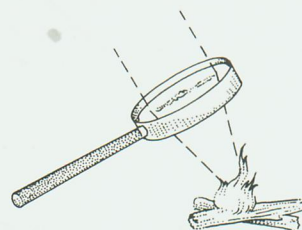
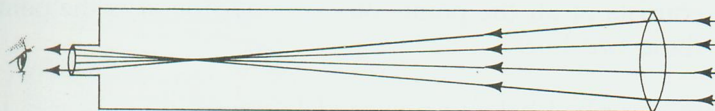
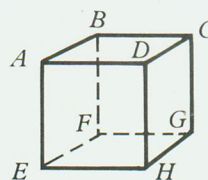
10. It is important to notice that three points may be collinear even though the lines may not be drawn. Name sets of three collinear points in this figure.
11. There is a line through each pair of points even though it may not have been drawn. Name two such lines in this figure.
12. Name three different lines which, if drawn, would be parallel to $\overleftrightarrow{BC}$.
13. Name three different lines which, if drawn, would be parallel to $\overleftrightarrow{EF}$.
14. Name four lines connecting labeled points that are concurrent at the center of the figure.
15. Points A , B , C , and D on this cube are coplanar. How many different sets of four coplanar points on the cube can you list?
16. Lines are often used to describe (or model) physical reality.
 - a. Starting a fire with a magnifying glass
 - b. Light coming from a flashlight
 - c. Using a refracting telescope



(Exs. 10, 11)



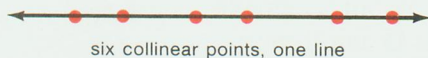
(Exs. 12-14)



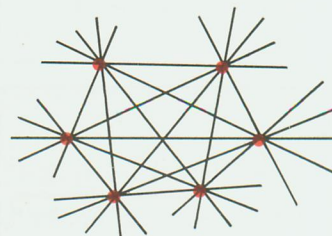
17. Can four lines be drawn so that they intersect in one point? Two points? Three points? Four points? Five points? Six points? More than 6 points? Show drawings for each.

PROBLEM SOLVING

How many lines do six points determine if one line is drawn through each pair of points?



Experiment! See if six points can be arranged to determine six lines. Arrange six points to determine seven lines, eight lines, nine lines, . . . 14 lines.



six points, no three
collinear, fifteen lines

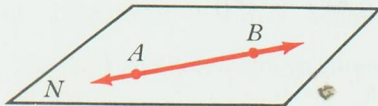
1-3 Some Basic Geometric Figures

Since lines, planes, and space are thought of as *sets of points* it is useful to define geometric figures in terms of sets and points. A *plane figure* is a figure with all points in one plane, but not all on one line. A *space figure* has points not all in a single plane.

First, we review some basic ideas about sets.

Subset. If every element of a first set is also in a second set, the first set is a *subset* of the second.

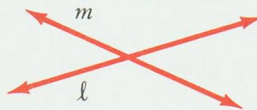
Example



Line AB is a *subset* of plane N .

Union. The *union* of two or more sets is a set that contains all the elements of these sets.

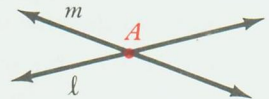
Example



The *union* of lines l and m consists of all the points on these two lines.

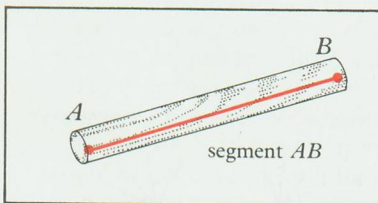
Intersection. The *intersection* of two sets is the set that contains those elements common to both sets.

Example



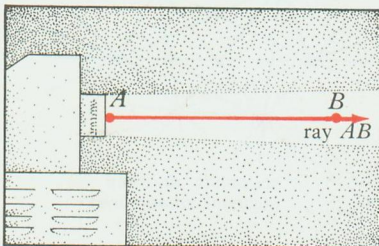
The intersection of line l and line m is the point A .

Some basic geometric figures are described below using models, symbols, and definitions.



A and B are endpoints.

We write: $\overline{AB}$



A is the endpoint.

We write: $\overrightarrow{AB}$

Definition 1-7

A **segment**, $\overline{AB}$, is the set of points A and B and all the points between A and B .

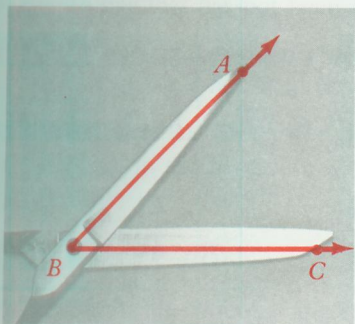
Definition 1-8

A **ray**, $\overrightarrow{AB}$, is a subset of a line that contains a given point A and all points on the same side of A as B .

Physical Model,
Figure

Description, Symbol

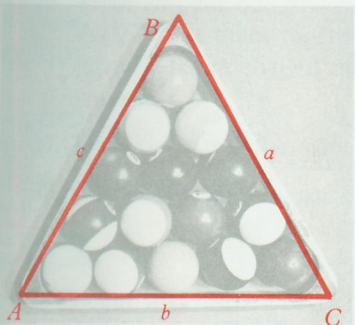
Definition



B is the *vertex*. $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are the *sides*. The interior of $\angle ABC$ is the intersection of the points on A 's side of $\overleftrightarrow{BC}$ with the points on C 's side of $\overleftrightarrow{AB}$.

Definition 1-9

An **angle** is the union of two noncollinear rays which have the same endpoint.

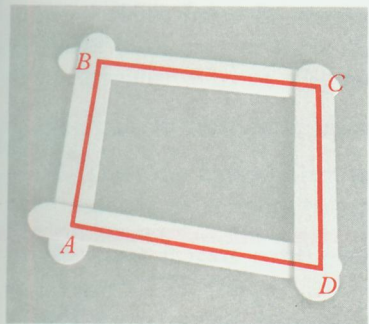


A , B , and C are *vertices*. $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are *sides*.

We write: $\triangle ABC$

Definition 1-10

A **triangle** is the union of three segments determined by three noncollinear points.

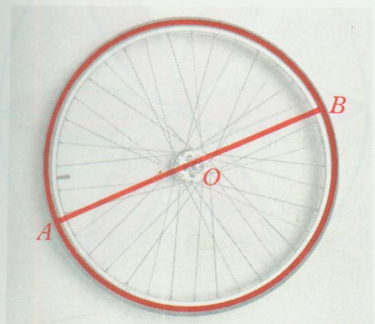


A , B , C , and D are *vertices*. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$ are *sides*.

We write: quadrilateral $ABCD$

Definition 1-11

A **quadrilateral** is the union of four segments determined by four points, no three of which are collinear. The segments intersect only at their endpoints.



Points A and B are on the circle. Point O is the center of the circle. $\overline{AB}$ is a *diameter* of the circle. $\overline{OB}$ is a *radius* of the circle.

We say: circle O

We write: $\odot O$

Definition 1-12

A **circle** is the set of all points in a plane that are a fixed distance from a given point in the plane.

EXERCISES

A.

Make six drawings of the line shown. Shade heavily each of the following figures, one on each drawing.

1. $\overline{BC}$ 2. $\overrightarrow{BD}$ 3. $\overleftrightarrow{CA}$ 4. $\overline{AD}$ 5. $\overrightarrow{BC}$ 6. $\overleftrightarrow{DB}$



(Exs. 1-6)

Draw and label an appropriate figure for each exercise 7-12.

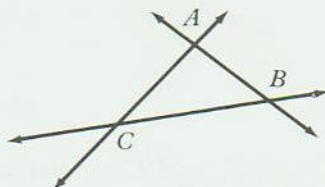
7. $\angle ABC$ 8. $\angle XYZ$ 9. $\triangle DEF$ 10. $\angle A$ 11. $\overrightarrow{BA}$ 12. $\overline{CD}$

In exercises 13-15 choose the two symbols that name the same angle.

13. $\angle ABC$, $\angle CAB$, $\angle CBA$

14. $\angle CAB$, $\angle BAC$, $\angle CBA$

15. $\angle ACB$, $\angle CAB$, $\angle BCA$



(Exs. 13-15)

16. $\triangle ABC$ and $\triangle BAC$ are two names for the triangle shown. There are four other names for this triangle. How many of them can you write?



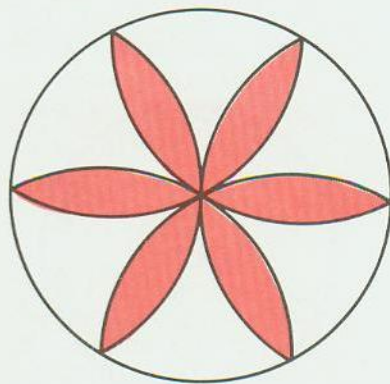
(Ex. 16)

17. Use a compass to draw a circle. Label the center, a radius, and a diameter of the circle. Write the name of the circle.
18. Choose points A , B , C , and D on the circle and draw quadrilateral $ABCD$.

Activity

To make this design draw a circle. Then, with compass opening equal to the radius and with compass tip placed at equal intervals around the circle, draw the arcs.

Complete a design like this. Then make and color some other designs, using both circles and segments, that involve the procedure described above. You may wish to have a ruler-compass design contest.



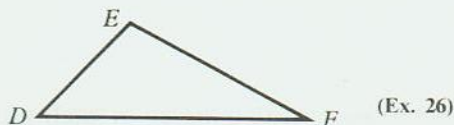
19. Name with symbols the four lines drawn in this figure. Name a line that has not been drawn.
20. Name eight segments which have been drawn. Name several which have not been drawn.

In exercises 21-24, choose the two symbols which name the same set in the figure.

21. $\overleftrightarrow{AB}$, $\overline{AB}$, p 22. $\overleftrightarrow{AE}$, $\overline{AC}$, q
 23. $\overleftrightarrow{BC}$, $\overline{CB}$, $\overline{BD}$ 24. $\overline{BC}$, $\overline{BD}$, $\overline{DB}$

25. Name six different angles in the figure.

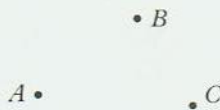
26. Trace and cut out two triangles like $\triangle DEF$. Put them together to form as many different shaped quadrilaterals as you can.



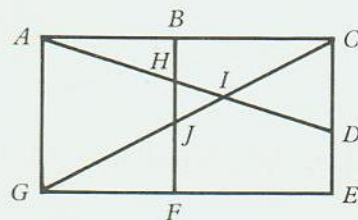
27. Does $\overline{CD}$ name the same segment as $\overline{DC}$? Why?
28. Does $\overrightarrow{CD}$ name the same ray as $\overrightarrow{DC}$? Why?

C.

29. Begin with three points as shown in the figure. Draw $\angle BAC$, $\angle ABC$, and $\angle ACB$. Does the resulting figure consist of three lines or three segments? Is it a triangle? Why or why not?



30. Name at least eight triangles in this figure.
31. Trace the figure. Then draw one more segment that will add exactly three more triangles.
32. Name at least eight quadrilaterals in this figure.



PROBLEM SOLVING

How can you glue six toothpicks together to form four triangles?

(Hint: Regardless of the size of the toothpicks, you will need to use plenty of space!)



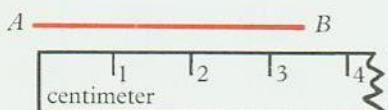
1-4 Segments and Angles—Congruence and Measurement

The man shown here is going to saw a board so that it will be 0.5 meters in length and will have an angled edge of 45° . First he has to measure.

Details of the basic properties of segment and angle measure will be presented on pages 72-73.



Length measure assigns a real number to each segment.



The *length* of $\overline{AB}$ is 3.5 cm.

We write: $AB = 3.5$

We have a special way to describe two segments that are the same length.



We say: $\overline{AB}$ is congruent to $\overline{CD}$.

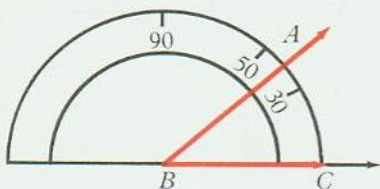
We write: $\overline{AB} \cong \overline{CD}$

We sometimes mark each segment to show that they are congruent.

Definition 1-13

Two **segments** are **congruent** if they have the same length.

Angle measure assigns a real number between 0 and 180 to each angle.

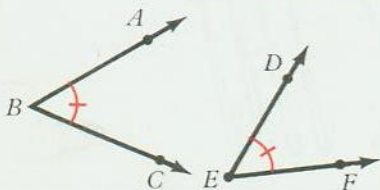


The *degree measure* of $\angle ABC$ is 40.

We write: $m\angle ABC = 40$

We sometimes write that $\angle ABC$ has a measure of 40° .

We have a special way to describe two angles that have the same measure.



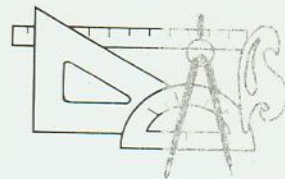
We say: $\angle ABC$ is congruent to $\angle DEF$.

We write: $\angle ABC \cong \angle DEF$

Definition 1-14

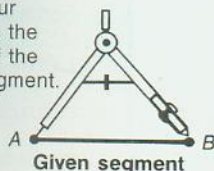
Two **angles** are **congruent** if they have the same measure.

A draftsman uses a variety of tools and drawing techniques to draw accurate plans for building projects. In geometry we will learn to use two tools—the unmarked ruler and the compass—to make special types of drawings called *constructions*. The two constructions described below use the idea of congruence defined on page 20.



Construction 1. Construct a segment congruent to a given segment. (Copy a segment.)

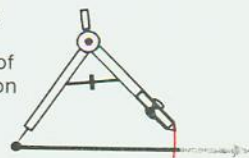
1. Open your compass the length of the given segment.



2. Draw a ray longer than the given segment.

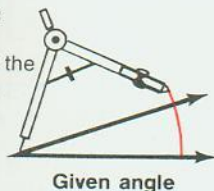


3. Use the same compass to mark a copy of the segment on the ray.



Construction 2. Construct an angle congruent to a given angle. (Copy an angle.)

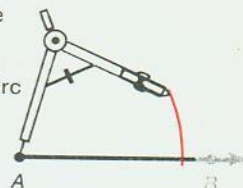
1. Draw an arc intersecting both rays of the given angle.



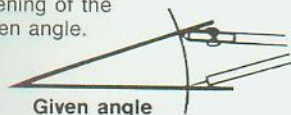
2. Draw a ray to serve as one side of the copy.



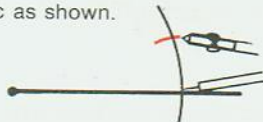
3. With the same compass opening as in (1), draw an arc crossing the ray.



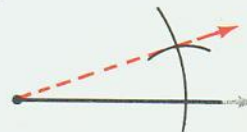
4. Open your compass to measure the opening of the given angle.



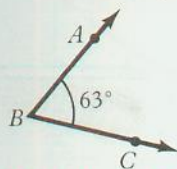
5. Use the same opening as in (4) and draw an arc as shown.



6. Draw the second side to complete the copy of the given angle.

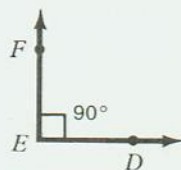


The three different types of angles are defined below. Use Construction 2 to construct angles congruent to each angle shown.



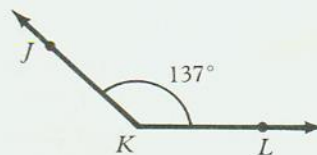
Definition 1-15

An **acute angle** is an angle with measure less than 90° .



Definition 1-16

A **right angle** is an angle with measure of 90° .



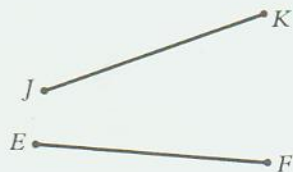
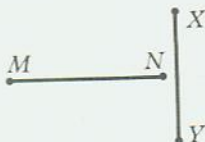
Definition 1-17

An **obtuse angle** is an angle with measure greater than 90° .

EXERCISES

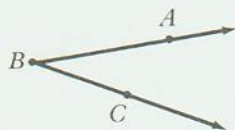
A.

- Use your centimeter ruler to find the length of this segment. $AB = ?$
- Draw segments with the following lengths. Use an unmarked ruler and a compass to construct segments congruent to each segment you draw.
 $CD = 8 \text{ cm}$ $EF = 5.6 \text{ cm}$ $GH = 7.5 \text{ cm}$
- Write a statement using $\cong$ or $\not\cong$ (not congruent) for the three sets of segments shown.



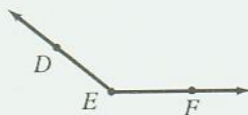
Trace these angles on your paper. Use your protractor to find their measures. Extend the sides if necessary.

4.



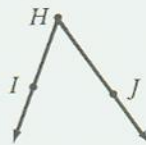
$$m\angle ABC = ?$$

5.



$$m\angle DEF = ?$$

6.



$$m\angle IHJ = ?$$

7.



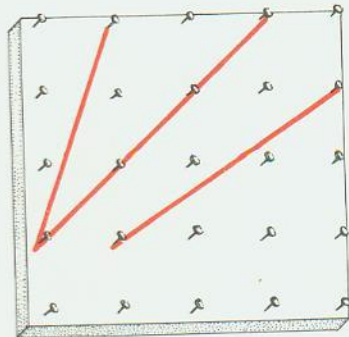
$$m\angle JKL = ?$$

- Classify the angles in exercises 4-7 as acute, right, or obtuse.
- Use an unmarked ruler and a compass to construct angles congruent to each of the angles in exercises 4-7.

Activity

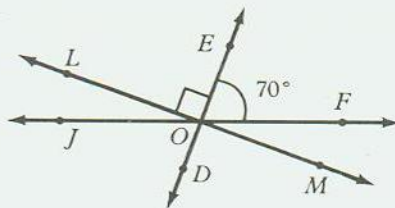
Begin with a 5×5 square array of dots. Two dots are endpoints of a segment. Two segments with a common endpoint determine an angle.

- How many different lengths of segments can be drawn on a 5×5 square array?
- How many different sizes of angles can be drawn on a 5×5 square array?



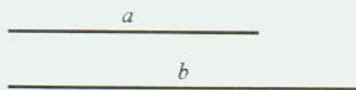
B.

10. Name four right angles in the figure.
11. Name four acute angles.
12. Name four obtuse angles.



Segments of lengths a and b are shown.

13. Construct a segment of length $2a$.
14. Construct a segment of length $a + b$.
15. Construct a segment of length $b - a$.



Angles with measures x and y are shown.

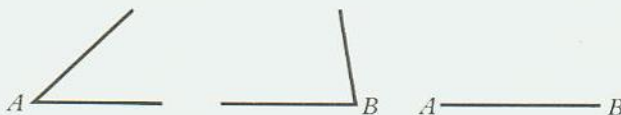
16. Construct an angle with measure $x + y$.
17. Construct an angle with measure $x - y$.
18. Construct an angle with measure $3y$.



19. A plane is flying in a southeasterly direction. Through how many degrees does it turn when it changes course to South-South West?

C.

20. Construct $\triangle ABC$ with side $\overline{AB}$ and angles A and B .



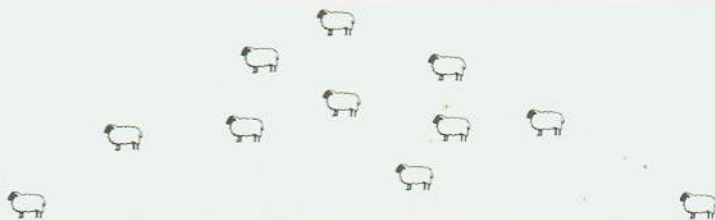
21. Follow these instructions to divide segment $\overline{AB}$ into three congruent segments.

- a. Draw a ray from point A and mark off three congruent segments on it.
- b. Draw $\overline{EB}$.
- c. Use Construction 2 to copy $\angle AEB$ at D and again at C .
- d. The sides of the copied angles divide $\overline{AB}$ into three congruent segments.



PROBLEM SOLVING

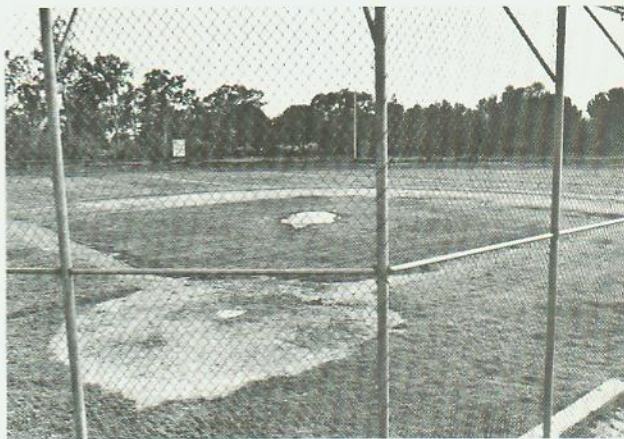
A farmer wishes to separate these 11 sheep by building eleven pens with exactly four straight fences. How can this be done? (The fences may cross.)



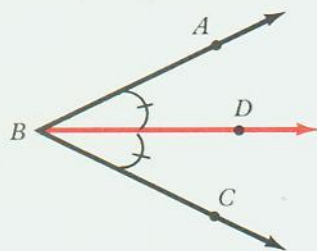
1-5 Segment and Angle Bisectors

In a baseball diamond, second base is the same distance from both foul lines. Home plate, first base, second base, and third base are at the corners of a square. Is the pitcher's mound at the point midway between

- home plate and second base?
- first base and third base?
- neither of these?



Laying out a baseball diamond involves the ideas defined below and the construction procedures described on page 25.



Ray $\overrightarrow{BD}$ is the *bisector* of $\angle ABC$. The points on $\overrightarrow{BD}$ are equal in distance from the sides of $\angle ABC$.

Definition 1-18

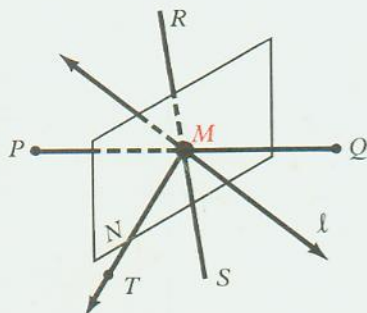
The **bisector of an angle** ABC is a ray $\overrightarrow{BD}$ in the interior of $\angle ABC$ such that $\angle ABD \cong \angle DBC$.



Point C is the *midpoint* of $\overline{AB}$.

Definition 1-19

The **midpoint of a segment** is a point C between A and B such that $\overline{AC} \cong \overline{CB}$.



$\overline{RS}$, $\overrightarrow{MT}$, line l , and plane N all intersect $\overline{PQ}$ at the midpoint M , and are bisectors of $\overline{PQ}$.

Definition 1-20

A **bisector of a segment** is any point, segment, ray, line, or plane that contains the midpoint of the segment.

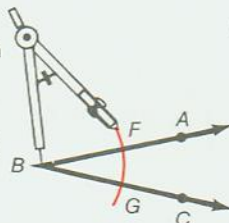
Methods for bisecting an angle and a segment are given below.

Construction 3. Bisect an angle.

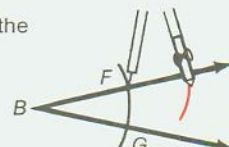
1. Given angle ABC .



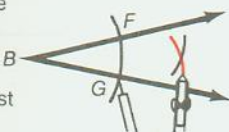
2. With B as center, draw an arc that intersects both sides of the angle, at F and G .



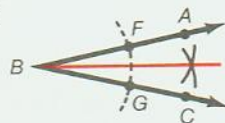
3. With F as center, draw an arc in the interior of the angle.



4. With G as center, and the same opening as in (3) draw an arc that crosses the first arc.

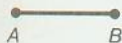


5. Connect B and the point of arc intersection to produce the bisector of the angle.

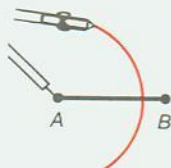


Construction 4. Bisect a segment.

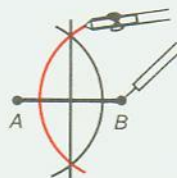
1. Given line segment AB .



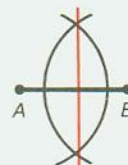
2. With A as center and opening greater than half of AB , draw a semicircular arc.



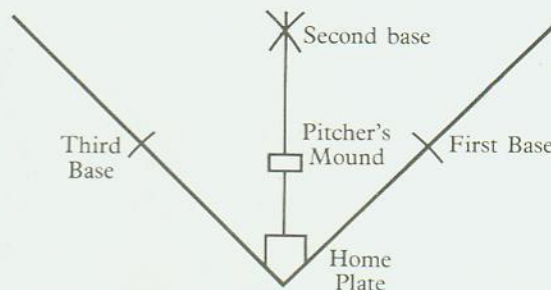
3. With B as center and the same opening as in (2), draw a semicircular arc that intersects the first arc.



4. Connect the two points of intersection to complete the construction of the bisector of AB .



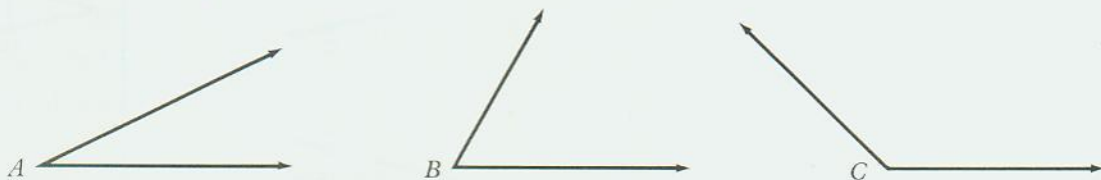
This diagram shows how bisecting an angle helps to lay out a baseball diamond. Notice that the pitcher's mound is on the angle bisector 60 feet 6 inches from home plate. It is not on a line from first to third base. Also, it is not at the midpoint of the line from home to second base.



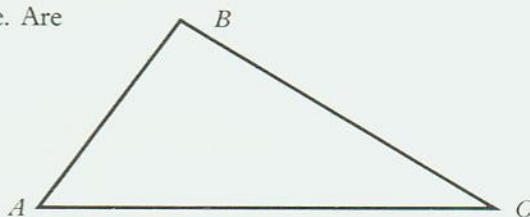
EXERCISES



1. Draw a segment $\overline{AB}$. Bisect $\overline{AB}$.
2. Draw a segment $\overline{AB}$. Construct a point N such that $AN = \frac{1}{4}AB$.
3. What other fractions of $\overline{AB}$ can you construct?
4. Draw angles the approximate sizes of angles A , B , and C .
Construct the angle bisectors.



5. Trace the triangle. Bisect each angle. Are the angle bisectors concurrent?

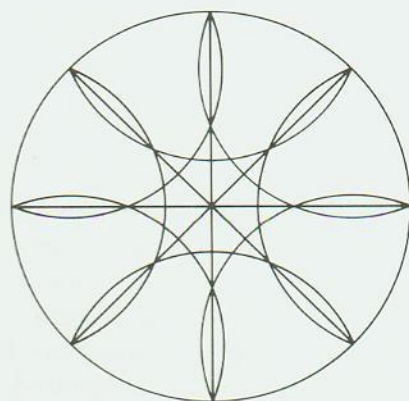
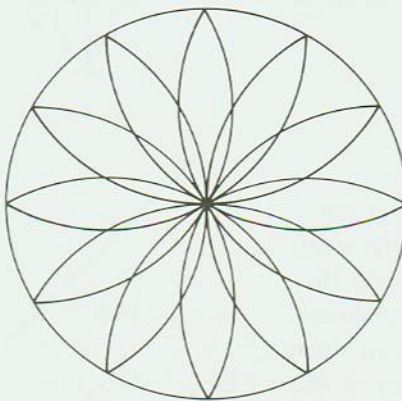


6. The bisector of $\angle XYZ$ is $\overrightarrow{YT}$. Write the names of the congruent angles that are formed.

Activity

To construct these designs, you need to bisect angles.

1. Construct one of these designs and color it.
2. Use a compass and straightedge to construct a design of your own which requires angle bisection.

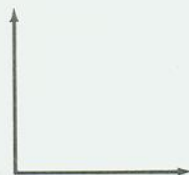
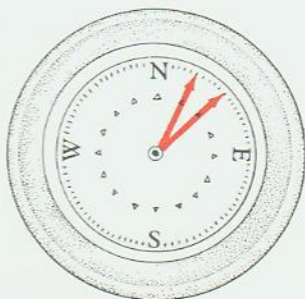




Use constructions to complete exercises 7-10. You may copy right angle ABC if you wish. Do not use a protractor.

7. Construct a 45° angle. 8. Construct a $22\frac{1}{2}^\circ$ angle.
 9. Construct a 135° angle. 10. Construct a $67\frac{1}{2}^\circ$ angle.
 11. Draw two acute angles. Call them $\angle J$ and $\angle K$. Construct a third angle that has measure $\frac{1}{2}(m\angle J + m\angle K)$. Do not use a protractor.

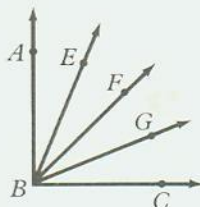
12. Draw the four directions found on a compass. Use a straightedge and compass to construct a line pointing in the direction North-North East.



(Exs. 7-10)



13. Construct a $112\frac{1}{2}^\circ$ angle. 14. Construct an $82\frac{1}{2}^\circ$ angle.
 15. Construct a $157\frac{1}{2}^\circ$ angle. 16. Construct a $97\frac{1}{2}^\circ$ angle.
 17. In the diagram, $\overrightarrow{BF}$ bisects $\angle EBG$, $m\angle ABC = 90$,
 $m\angle ABE = 20$, $m\angle GBC = 24$.
 What is $m\angle ABF$?



18. Construct a segment with length $4CD - \frac{1}{2}AB$.



PROBLEM SOLVING

A *rusty compass* is one that is stuck and always has the same opening.

A *collapsible compass* is one that springs back together as soon as it is lifted from the paper.

1. Bisect $\overline{AB}$ using:

- a. a collapsible compass b. a rusty compass with opening from C to D.

$\dot{C}$

$\dot{D}$



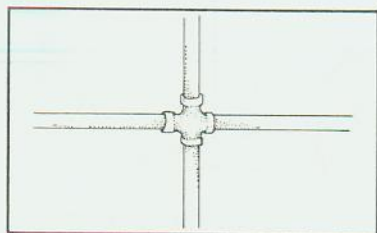
2. Bisect an angle using:

- a. a collapsible compass b. a rusty compass.

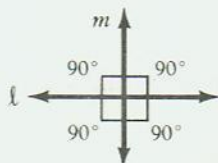
1-6 Perpendicular Lines and Planes

There are many examples of perpendicular lines and planes in our everyday life. We use some of these examples in the following definitions.

Physical Model



Figure, Description



l is perpendicular to m .
We write: $l \perp m$

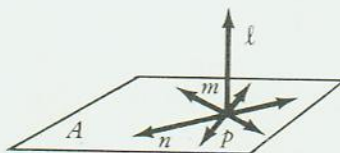
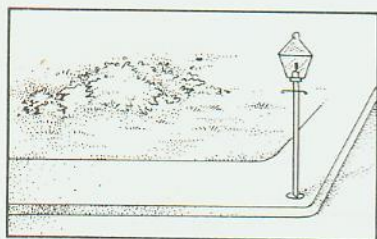
Definition

Definition 1-21

Two lines are **perpendicular** if they intersect to form congruent right angles.

On the basis of simple statements that can be proved, we will interpret this definition of perpendicular to include these ideas:

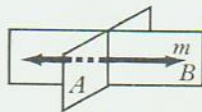
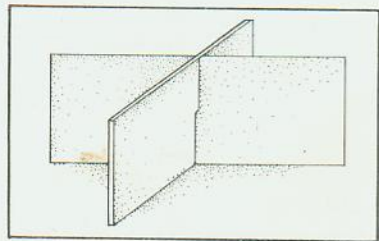
1. When two lines are perpendicular, all angles formed are 90° angles (right angles), and they are congruent.
2. When two lines intersect to form one, two, or three 90° angles (right angles), the lines form four right angles and are perpendicular.
3. When two lines intersect to form a pair of congruent angles with a common side, the lines are perpendicular.



Line l is perpendicular to lines m , n , p , etc., so line l is perpendicular to plane A .

Definition 1-22

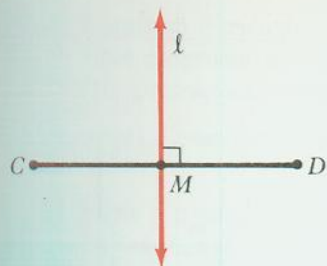
A **line is perpendicular to a plane** if it is perpendicular to each line in the plane that intersects the line.



Line m in plane B is perpendicular to plane A , so plane B is perpendicular to plane A .

Definition 1-23

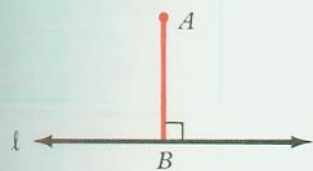
Two **planes are perpendicular** if there is a line in one plane that is perpendicular to the other plane.



l is the perpendicular bisector of $\overline{CD}$.

Definition 1-24

The **perpendicular bisector of a segment** is a line perpendicular to the segment and which contains its midpoint.



AB is the distance from point A to line l .

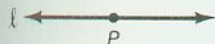
Definition 1-25

The **distance from a point to a line** is the length of the segment drawn from the point perpendicular to the line.

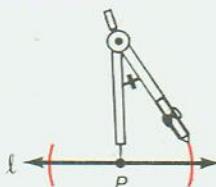
In Construction 4, we actually constructed the perpendicular bisector of a segment. Here are two more important constructions that involve perpendicular lines.

Construction 5. Construct a perpendicular to a line through a given point on the line.

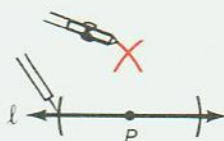
1. Given a line l and a point P on l .



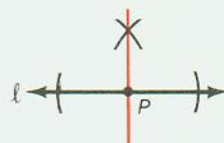
2. Draw arcs on each side of P .



3. Draw crossing arcs above line l .



4. Draw the perpendicular to line l through P .

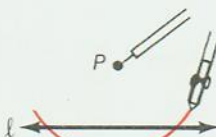


Construction 6. Construct a perpendicular to a line through a given point not on the line.

1. Given a line l and a point P not on l .



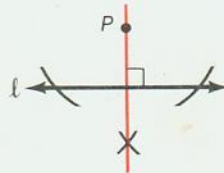
2. Draw two arcs cutting line l .



3. Draw two crossing arcs below line l .



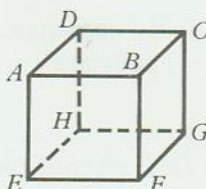
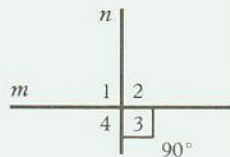
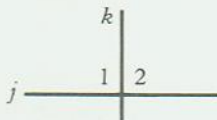
4. Draw the perpendicular to line l through P .



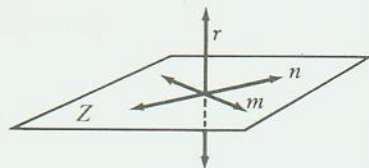
EXERCISES

A.

- $\angle 1 \cong \angle 2$. What can you conclude about lines j and k ?
- $m \angle 3$ is 90° . What can you conclude about lines m and n and angles 1, 2, and 4?
- Draw a segment $\overline{PQ}$. Construct the perpendicular bisector of $\overline{PQ}$.
- Draw a line ℓ . Construct a line perpendicular to ℓ that contains point A on ℓ .
- Draw a line m and a point P not on m . Construct a line perpendicular to m through P .
- Name four lines that are perpendicular to plane $ABCD$.
- Name four pairs of perpendicular planes.



(Exs. 6, 7)



- Line r is perpendicular to plane Z . What does this tell you about lines r , m , and n ?

B.



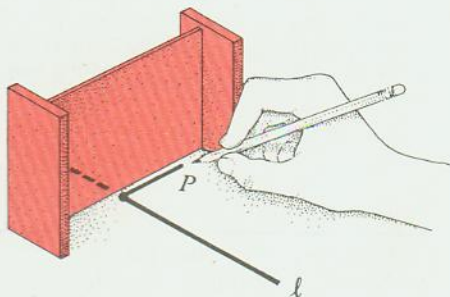
- Construct a rectangle with sides congruent to $\overline{AB}$ and $\overline{CD}$.



Activity

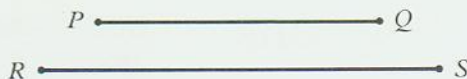
These directions show how to use a MIRA or a sheet of colored plastic to construct a perpendicular to a line through a point not on the line. (Construction 6) Show how to do Constructions 3, 4, and 5 in this chapter using the MIRA or plastic.

Place the MIRA with the drawing edge next to P in such a way that the visual image of the half of line ℓ in front of the MIRA is exactly on the half of line ℓ that is behind the MIRA. Draw along the drawing edge to produce the perpendicular to line ℓ through point P .





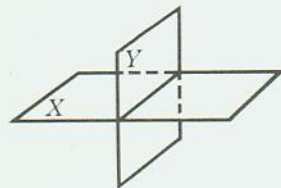
10. Construct a triangle with a 45° angle and sides congruent to segments $\overline{PQ}$ and $\overline{RS}$. (No protractors please!)



11. Apply Construction 5 to construct a square with side $\overline{AB}$.



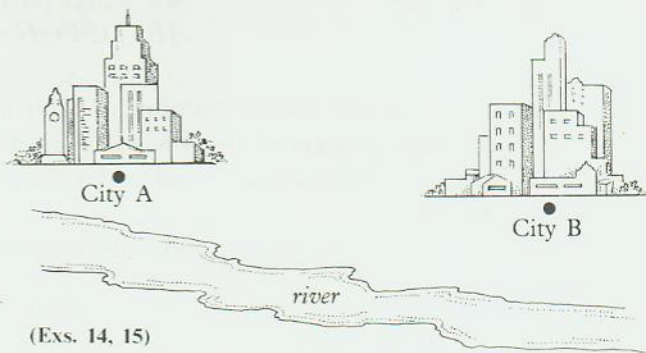
12. Draw a triangle ABC . Apply Construction 6 to construct a point D on $\overline{BC}$ so that $\overline{AD}$ is perpendicular to $\overline{BC}$.
13. Is plane X perpendicular to plane Y ? According to the definition of perpendicular planes, what information is needed to be sure that the planes are perpendicular?



C.

14. Two cities need additional water. They decide to build a water treatment plant along a nearby river and pipe water from the plant to each city. Each city will pay the cost of laying the pipe from the plant to its city. The plant should be located an equal distance from the two cities.
15. Another proposal for the water treatment plant location in Exercise 14 calls for the cities to share the cost of the entire pipeline equally. Under this plan the total length of the pipeline should be kept to a minimum. Where should the treatment plant be located? Trace the map and construct the desired location of the plant.

Trace the map shown and determine through construction the point where the plant should be located under this proposal.



PROBLEM SOLVING

Shown are four identical cubes from each of which one or more small, also identical cubes have been removed.



A



B



C

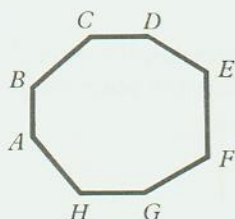


D

Compare each pair of figures: A & B, A & C, A & D, B & C, B & D, and C & D. Which one pair could be the same (depending upon the hidden back corner)?

1-7 Polygons

Geometric figures made up of straight line segments are common in the world. Such figures are called *polygons*.



This polygon has eight sides. Points $A, B, C, D, E, F, G,$ and H are its vertices. Each segment of the polygon is called a side.

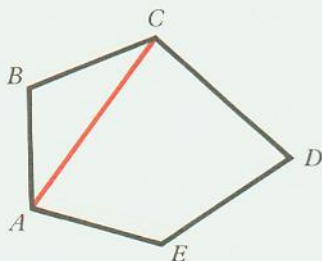
We write: polygon $ABCDEFGH$

Definition 1-26

A **polygon** is the union of segments meeting only at endpoints such that (1) at most two segments meet at one point, and (2) each segment meets exactly two other segments.

Polygons are named by the number of their sides. For example, triangle—3 sides; quadrilateral—4 sides; pentagon—5 sides; hexagon—6 sides; heptagon—7 sides; octagon—8 sides. A polygon with n sides can be called an **n-gon**.

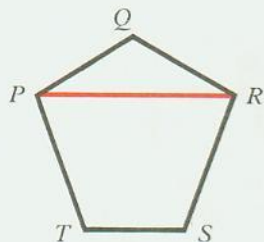
The following definitions give more information about polygons.



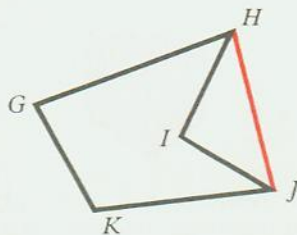
The end points of $\overline{AC}$ are non-consecutive vertices of polygon $ABCDE$. $\overline{AC}$ is one of the *diagonals* of the polygon.

Definition 1-27

A **diagonal of a polygon** is a segment joining any two nonconsecutive vertices of the polygon.



Every diagonal of this polygon, like $\overline{PR}$, is in the interior of the polygon. $PQRST$ is a convex polygon.

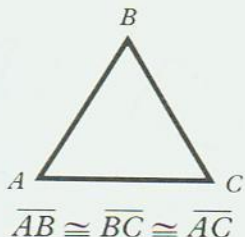
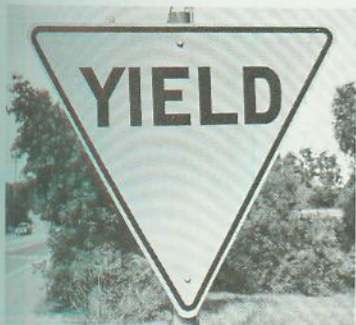


At least one of the diagonals of this polygon is not in its interior. $GHIJK$ is not a convex polygon.

Definition 1-28

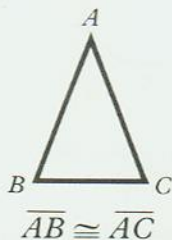
A **polygon** is **convex** if all of the diagonals of the polygon are in the interior of the polygon.

Triangles with congruent sides have special names.



Definition 1-29

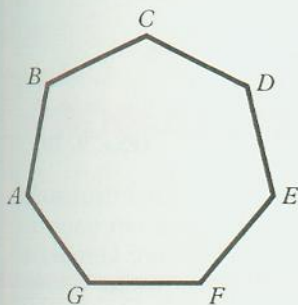
An **equilateral triangle** is a triangle with all sides congruent to one another.



$\angle A$ is called the *vertex angle*.
 $\angle B$ and $\angle C$ are called *base angles*.

Definition 1-30

An **isosceles triangle** is a triangle with two sides congruent to one another.



Some polygons have properties that make them *regular polygons*.

All sides are equal in length.
 All angles are equal in measure.

Definition 1-31

A **regular polygon** is a polygon with all sides congruent to each other and all angles congruent to each other.

$ABCDEFGH$ is a regular polygon.



EXERCISES

A.

In exercises 1–4 select the figure that is not a polygon. Explain why it is not a polygon.

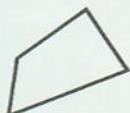
1.



a.



b.



c.

2.



a.



b.



c.

3.



a.



b.



c.

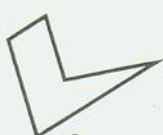
4.



a.



b.



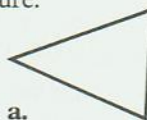
c.

5. Which figures above are convex polygons? For example, figure 1c is a convex polygon.

6. Draw a convex octagon. Draw a diagonal of this figure.

7. Draw a non-convex octagon.

8. Identify each triangle as equilateral, isosceles, or neither. Use a ruler.



a.



b.



c.

9. Which of these polygons are regular polygons?



a.



b.



c.



d.

(Exs. 9, 10)

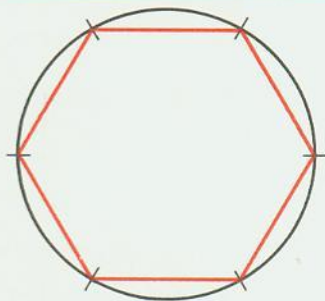
10. Draw as many diagonals as possible for each polygon.

Activity

The compass and straightedge construction method described in the activity on page 18 can be used to construct a regular hexagon.

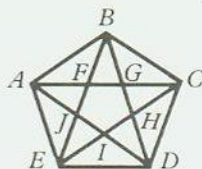
Combine this method with bisecting angles and constructing perpendicular bisectors to construct:

- a regular 12-gon
- a regular octagon
- a regular 16-gon.



B.

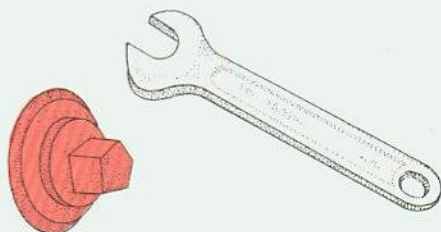
11. $ABCDE$ is a regular pentagon. Name as many isosceles triangles as you can. (If a triangle looks isosceles, you may assume that it is.)



12. Some letters of the alphabet can be drawn as a polygon, and others cannot. Draw as many polygons in the shape of letters of the alphabet as you can.

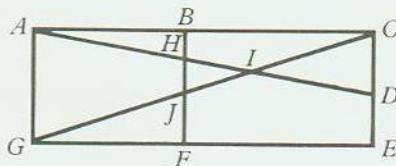


13. An open-faced wrench has parallel edges. What do you know about the number of edges on a nut that this wrench fits?
14. The valve stem on the top of a fire hydrant is usually in the shape of a regular pentagon rather than the usual shape of a regular hexagon. Why do you think this is done?



C.

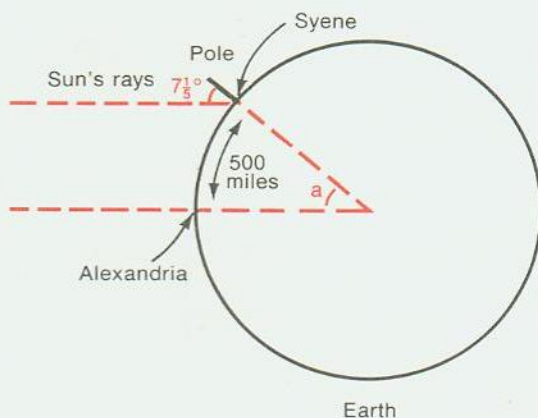
15. The figure on the right includes examples of all polygons from 3-gons to 10-gons. Find and name an example of each.



16. Use a protractor and a ruler to draw an octagon with sides of five different lengths all of whose vertex angles measure 135° .

PROBLEM SOLVING

Erastosthenes (275 B.C.) calculated the distance around the earth using an ingenious method. He assumed the rays from the sun to be parallel and found that when the sun was directly overhead in Alexandria, its rays made an angle of $7\frac{1}{5}^\circ$ with a vertical pole 500 miles away in Syene. He assumed the central angle a to be $7\frac{1}{5}^\circ$ also. He reasoned that the ratio of central angle a to 500 miles would be equal to the ratio of the total number of degrees in a circle to the complete distance around the earth. Can you set up the proportion and find the distance?



Important Ideas—Chapter 1

Terms

Point (p. 10)
 Line (p. 10)
 Plane (p. 11)
 Space (p. 11)
 Collinear points (p. 13)
 Coplanar (p. 13)
 Intersecting lines (p. 13)
 Parallel lines (p. 13)
 Concurrent lines (p. 13)
 Segment (p. 16)
 Ray (p. 16)
 Angle (p. 17)
 Triangle (p. 17)
 Quadrilateral (p. 17)
 Circle (p. 17)
 Congruent segment (p. 20)
 Congruent angle (p. 20)

Acute angle (p. 21)
 Right angle (p. 21)
 Obtuse angle (p. 21)
 Bisector of an angle (p. 24)
 Midpoint of a segment (p. 24)
 Bisector of a segment (p. 24)
 Perpendicular lines (p. 28)
 Line perpendicular to a plane (p. 28)
 Perpendicular planes (p. 28)
 Perpendicular bisector (p. 29)
 Distance from a point to a line (p. 29)
 Convex polygon (p. 32)
 Diagonal of a polygon (p. 32)
 Convex polygon (p. 32)
 Equilateral triangle (p. 33)
 Isosceles triangle (p. 33)
 Regular polygon (p. 33)

Constructions

Copy a segment (p. 21)
 Copy an angle (p. 21)
 Bisect an angle (p. 25)
 Bisect a segment (p. 25)

Construct a perpendicular to a line through a given point on the line (p. 29)
 Construct a perpendicular to a line through a given point not on the line (p. 29)

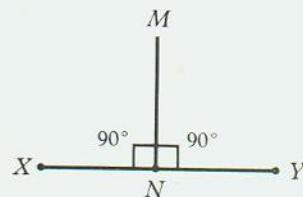
Chapter 1—Review

1. Name objects that illustrate the following:

- a. point b. plane c. parallel lines
d. intersecting lines e. polygon

2. Mark the following as true or false.

- a. A laser beam is a better example of a line than a ray.
b. $\overline{MN}$ is not perpendicular to $\overline{XY}$ since only two right angles are formed.



c. Point C is on $\overrightarrow{AB}$.



d. Point C is on $\overleftrightarrow{AB}$.

e. A radius is a ray.

f. Two segments are congruent if they have the same length.

3. How many endpoints does a line have? a ray? a segment?

4. Is $\overline{AB}$ the same as $\overline{BA}$? Why or why not?

5. Construct a 135° angle.

6. Draw an obtuse angle and construct its bisector. Are the resulting angles acute, right, or obtuse?

7. Draw a triangle ABC . (Make it fairly large.) Construct the midpoint of each of the three sides.



8. Construct a segment that is congruent to $\overline{AB}$.

9. Construct a segment that has length $\frac{1}{4}AB$.

• P

10. Copy $\overline{CD}$ and P on your paper. Construct the line through P perpendicular to $\overline{CD}$.



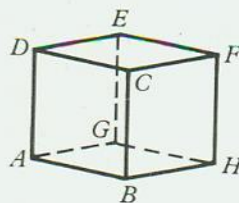
In exercises 11–14 use the figure of a cube to identify the following:

11. Two parallel planes.

12. A line perpendicular to plane $EFHG$.

13. A line parallel to plane $CDEF$ but not perpendicular to plane $ABCD$.

14. The intersection of plane $BCEG$ and plane $BCFH$.



Chapter 1—Test

- Name objects that illustrate the following:
 - line
 - concurrent lines
 - perpendicular lines
 - regular polygon
 - space.

- Mark the following as true or false.

- Two points are always collinear.
- If two angles are congruent then they are both right.
- Point D is on $\overrightarrow{AB}$.
- Point D is on $\overrightarrow{AC}$.
- A diameter of a circle is a line.
- An isosceles triangle must have three congruent sides.

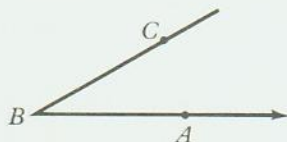


- Is $\overrightarrow{AB}$ the same as $\overrightarrow{BA}$? Why or why not?
- If two lines are parallel, how many points do they have in common?

- Draw a non-convex quadrilateral.

- Draw a triangle ABC . Construct the angle bisector of each angle.

- Construct an angle that is congruent to $\angle ABC$.



- Without using a protractor, construct four right angles with a common vertex.

- Construct a segment congruent to $\overline{AB}$.



(Exs. 9, 10)

- Construct the midpoint of $\overline{AB}$.

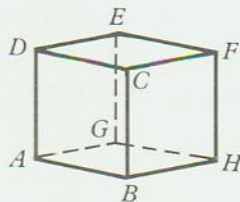
In exercises 11–14 use the figure of a cube to identify the following:

- A line parallel to plane $ABHG$.

- A plane perpendicular to $\overrightarrow{EG}$.

- A line that is not parallel and also not perpendicular to any face of the cube.

- The intersection of plane $ADFH$ and plane $CDEF$.



Problem Solving Techniques

Draw a Diagram

Problems can be fun to solve if you know ways to tackle them. There are several techniques for solving mathematical problems. One of these is to draw a diagram.

Example

On ray $\overrightarrow{AB}$, $AC = 5$ and $AB = 2$. Find BC . To help us solve the problem, we will draw a diagram.



From the diagram, we see that $BC = 3$.

PROBLEMS

Draw diagrams and then solve the following problems.

1. On ray $\overrightarrow{NP}$, $NP = 4$ and $PM = 6$. Find NM .
2. On ray $\overrightarrow{XY}$, $XY = 15$ and $YZ = 6$. Find ZX .
3. How many posts are needed to fence a rectangular field if the posts are to be 5 feet apart and the field is 20 feet by 30 feet?
4. Suppose a bug crawls up a pole two inches in two minutes, then down one inch in one minute, then up two inches in two minutes, and so on. At this rate, how long would it take the bug to climb a height of 10 inches?
5. A stairway has 10 steps, each one foot wide and one foot high. An ant starts at the bottom of the first step and travels straight up the stairway. How far had the ant traveled when it reached the top of the last step?
6. A city high school set up a cross country course through the streets of the city. From the starting point the runners ran 4 blocks east, then 6 blocks north, then 2 blocks west, then 2 blocks south, then 5 blocks west, then 3 blocks north, then 2 blocks west, then 8 blocks south, and 5 blocks east to the finish line. State the direction and the number of blocks from the starting point to the finishing point.

Geometry In Our World

Interior Design: Tessellations

An interior designer encounters examples of geometry when selecting fabric designs, floorcoverings, and wallpaper. Often a geometric concept called a *tessellation* is used in a design. A tessellation is an arrangement of polygons such that no polygons overlap and there are no gaps.

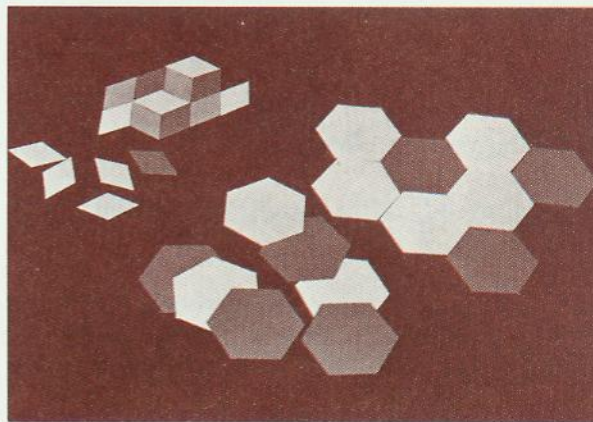
The kitchen shown here has a floor covering and tiled counters that are examples of tessellations.



1

Which polygons will tessellate (or *tile*) a flat surface?

By laying tracing paper over a square and repeatedly tracing it, you can show a “tiling” (or tessellation) of squares. Other tessellations are shown in the picture. Show on your tracing paper a portion of a tessellation using each figure below for which it is possible. Which two figures below *cannot* be used in a tessellation?



Parallelogram



Trapezoid



Kite



Any quadrilateral



Equilateral triangle



Isosceles triangle



Scalene triangle



Nonconvex quadrilateral



Heptagon



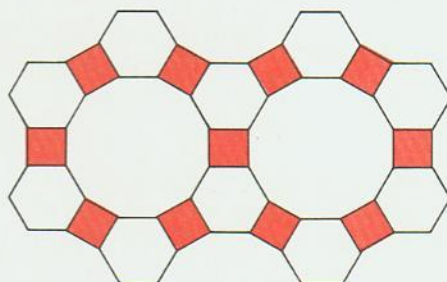
Pentagon



Hexagon

2 Which combinations of regular polygons will tessellate a flat surface?

Interesting floor patterns can be produced by forming tessellations that combine some of the regular polygons pictured below. The pattern shown here has a *square*, a *hexagon*, and a *dodecagon* surrounding each point. It is named using the numbers (4, 6, 12). These numbers show the number of sides of the figures used and the exact order of their arrangement around the point.



(4, 6, 12)

Trace the figures pictured here to show on your tracing paper a portion of each of the following tessellations. Color each in an interesting way!

(4, 8, 8)

(3, 4, 6, 4)

(3, 6, 3, 6)

(3, 3, 3, 4, 4)

(3, 12, 12)



Dodecagon



Hexagon



Triangle



Square



Octagon

